Positions, Regions, and Clusters: Strata of Granularity in Location Modelling

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Abstract. Location models are data structures or knowledge bases used in Ubiquitous Computing for representing and reasoning about spatial relationships between so-called smart objects, i.e. everyday objects, such as cups or buildings, containing computational devices with sensors and wireless communication. The location of an object is in a location model either represented by a *region*, by a coordinate *position*, or by a *cluster* of regions or positions. Qualitative reasoning in location models could advance intelligence of devices, but is impeded by incompatibilities between the representation formats: topological reasoning applies to regions; directional reasoning, to positions; and reasoning about set-membership, to clusters. We present a mathematical structure based on scale spaces giving an integrated semantics to all three types of relations and representations. The structure reflects concepts of granularity and uncertainty relevant for location modelling, and gives semantics to applications of RCC-reasoning and projection-based directional reasoning in location models.
Keywords: Spatial granularity, uncertainty, location model, pervasive computing, ubiquitous computing, scale space, qualitative spatial reasoning

1 Introduction

Location models are data structures or knowledge bases used in Ubiquitous Computing for representing and reasoning about spatial relationships between so-called *smart objects* or *artefacts*. Smart objects – everyday objects, such as the Mediacups [1] or smart buildings, which contain computational devices equipped with sensors and wireless communication – can react intelligently to their spatial environment and to each other through a location model [2]. Generally, three types of location models are distinguished, with each employing certain characteristic types of representation [9]:

- *Symbolic location models* are qualitative location models in which the relationships between named *regions* are represented.
- *Geometric location models* are quantitative location models in which the *position* is represented by means of measured coordinates.

– *Hybrid location models* contain symbolic as well as geometric location information. Additionally, compounds of heterogeneous artefacts can be represented by finite sets of regions and/or positions, which we will call *clusters* in the following. A complex large-scale artefact, such as a smart building consisting of many locations and devices, is an example of a cluster.

Qualitative spatial reasoning (QSR, cf. [3] for an overview) about locations would be an ideal reasoning technique to enable interesting new intelligent interaction methods for artefacts, as QSR offers a lightweight reasoning approach that can be implemented well even under resource restrictions, as found in autonomous robots [3] or in the small devices of Ubiquitous Computing. However, application of QSR for location models is currently impeded by seeming incompatibilies between the three formats:

– Regions can be compared well with mereotopological relations [13], but relations such as *overlap* and *proper part* are trivial for positions: two points are either identical or differ. For clusters as sets of positions and/or regions, mereotopological relations can be counterintuitive: are the parts of a cluster its elements or the parts of its elements? Are the two in some sense the same?
– Position representations can be compared qualitatively using directional calculi [10]. But relations such as north-of are considerably more difficult to define for regions [14, 17]: is a hallway surrounding a room north, west, east, or south of the room, or does it have all relations at the same time, or none? The same holds even more so for clusters.
– Clusters and their elements are related through the set-theoretical relation of membership and correpondingly definable relations, such as subset. Like mereotopological relations, set-theoretical notions are trivial for positions, either two positions are identical or different. Counterintuitively however, a cluster of one point and the point itself are not the same location in terms of set-theoretical relations. Set-theoretical notions can be applied to regions in a standard way by interpreting regions as point-sets [4].

The aim of this paper is to show that a set of widely used mereotopological and directional QSR-relations can be interpreted in terms of relations over sets of points in a scale space. The resulting structure gives an integrated semantics to all three types of representations found in location modelling in a way so as to reflect uncertainty and heterogeneity of location measurement in Ubiquitous Computing.

After a brief overview of related works (Sect. 2), we introduce the key notion of σ-points as uncertain coordinates in a scale space in Sect. 3. In Sect. 4, we then show how a set of well-known mereotopological [13] and directional relations [10] can be interpreted with respect to scale spaces. We discuss conclusions and open questions in Sect. 5.

2 Related Works

The mathematical structure we propose is built upon notions from scale-space theory, a theory of multi-scale analysis of signals and images used in computer vision research and signal processing [11]. We apply these notions in a novel way to link qualitative reasoning to uncertain quantitative measurements in Ubiquitous Computing. Furthermore, the work presented in this paper draws upon and generalizes related ideas from research on granularity, spatial databases, and diagrammatic reasoning.

When we use knowledge about spatial phenomena, a main characteristic we have to handle is the *spatial granularity* required for a task. Granularity can be understood as a parameter of the task that determines which objects and relations are applicable for the task. In [16], we proposed a mereological axiomatisation of spatial granularity based on size. We go beyond the concepts introduced in [16] as we also represent limitations of accuracy in location models. Coordinate information from location sensing is always subject to uncertainty and varies considerably with devices [6]. Layered multi-resolution *spatial databases* can store objects having different representations on multiple scales, so that each representation is associated with a certain range of scales [5, 12]. The idea of a scale parameter has also been applied for indexing in spatial databases [7] and to diagrammatic visual inference about interval relations [8].

In contrast to spatio-temporal databases and geographic information systems, which contain only quantitative location information in fixed coordinate systems, location models can contain a mixture of qualitative and quantitative spatial information without a common reference system. Each room in a smart building can have its own reference system. We generalise the idea of a scale-parameter in two respects: on the one hand, we address spaces of arbitrary dimension n, and on the other hand, we use the additional coordinate for representing uncertainty of location measurement. The generalised idea is to represent an n-dimensional measurement coordinate $\boldsymbol{m}$ together with a standard deviation σ describing the accuracy of the measurement device or computation that produced the coordinate value. We obtain an $n+1$-dimensional coordinate, a scale-dependent point, which maps to an n-dimensional range of uncertainty.

3 Scale-Dependent Positions, Regions, and Clusters

In order to account for uncertainty in location measurement devices, we interpret a scale-dependent $n+1$-dimensional coordinate $(\boldsymbol{m}, \sigma)$, consisting of an n-dimensional coordinate vector $\boldsymbol{m} \in \mathbb{R}^n$ and a scale $\sigma \in \mathbb{R}^+ = \{r \in \mathbb{R} \mid r > 0\}$, by a Gaussian function

$$G(\boldsymbol{x}; \boldsymbol{m}, \sigma) = \frac{1}{\sigma(2\pi)^{\frac{n}{2}}} \exp\left[-\frac{d(\boldsymbol{x}, \boldsymbol{m})^2}{2\sigma^2}\right] \tag{1}$$

that centres around the measured coordinate $\boldsymbol{m}$, with the standard deviation σ obtained from the specification of the measurement device. Here, $d(\boldsymbol{x}, \boldsymbol{y})$ is a distance function. Random observational error in location measurement can be represented in the σ-coordinate: when $\boldsymbol{m}$ has been measured, the actual coordinate is within a range δ of $\boldsymbol{m}$ with probability a, the accuracy of the

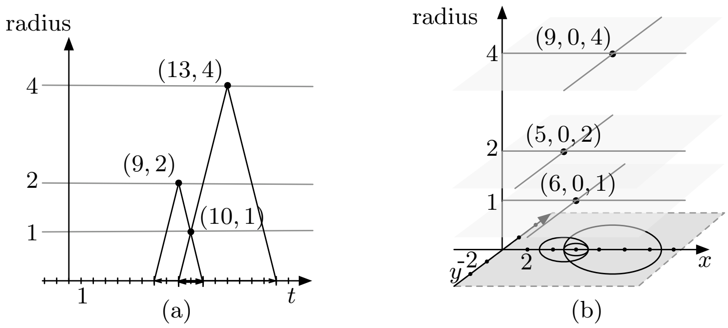

Fig. 1. Scale spaces: a) the interval from $t = 7$ to $t = 11$ with the centre at $t = 9$ and a radius of 2 (i.e., duration of 4) time units is represented as a point at the coordinate $(9, 2)$ in the 2-dimensional scale space of intervals (cf. [8]); b) the open disc $A = \{(x, y) \mid d((x, y), (5, 0)) < 2\}$ with centre at point $(5, 0)$ and radius 2 is represented as a point at the coordinate $(5, 0, 2)$ in a 3-dimensional scale space.

device:

$$\int \cdots \int_{d(\boldsymbol{x},\boldsymbol{m}) \leq \delta} G(\boldsymbol{x}; \boldsymbol{m}, \sigma) dx^n = a. \tag{2}$$

Using this relationship between a range δ and a corresponding accuracy a, an application can ensure its accuracy is a by performing its reasoning and calculations on the δ-sphere around $\boldsymbol{m}$ for which (2) holds. For any fixed accuracy a we can then derive a transformation function mapping uncertain coordinate information in $\mathbb{R}^n \times \mathbb{R}^+$ obtained from measurement to corresponding regions of uncertainty in $\mathbb{R}^n$. An application requiring a certain accuracy of, for instance, $a = 95\%$ can look up δ so that (2) holds. In the 1-dimensional case, for instance, we obtain that $\delta = 2\sigma$ for $a = 95\%$ and $\delta = \sigma$ for $a = 68\%$; that is, δ is a multiple $\lambda_a \sigma$ of σ, with λ_a depending on a. The $n + 1$ dimensional coordinate $\boldsymbol{e} = (x_1, \ldots, x_n, \sigma)$ of a position can thus be interpreted by the application as the n-sphere of radius $\lambda_a \sigma$ that has its centre at $\boldsymbol{x} = (x_1, \ldots, x_n)$.

In order to avoid confusion between the points of the n-dimensional underlying space $\mathbb{R}^n$ and points, lines, planes, etc. in the $n + 1$-dimensional multi-scale space $\mathbb{R}^n \times \mathbb{R}^+$, we continue to call the former *points*, *lines*, *planes*, etc. as before and call the latter *σ-points*, *σ-lines*, *σ-planes*, etc. below. For readability, we use variables $\boldsymbol{e}$, $\boldsymbol{e_1}$, $\boldsymbol{e_2}$ to range over σ-points and variables $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, etc. to refer to points.

Figure 1 illustrates the case of (a) 1-dimensional and (b) 2-dimensional position measurements from devices with different accuracies. For the 2-dimensional case, we obtain a set of discs of $\mathbb{R}^2$, with each disc specified by three coordinates: a centre $c \in \mathbb{R}^2$ and a radius $r \in \mathbb{R}^+$. In order to describe the translation between representations in the two spaces, we define a family of functions $f_a : \mathbb{R}^n \times \mathbb{R}^+ \to 2^{\mathbb{R}^n}$ mapping coordinates, in the $n + 1$-dimensional multi-scale space $\mathbb{R}^n \times \mathbb{R}^+$ to n-dimensional spheres in the underlying space $\mathbb{R}^n$ for an accuracy a. We define f_a using a projection function $c : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ yielding the centre of the sphere referred to by a σ-point, and functions $\sigma_a : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^+$ that yield the radius of the sphere for a given accuracy a. Then, the σ_a-sphere around the centre point $c(\boldsymbol{e})$ is the set of all points whose distance to $c(\boldsymbol{e})$ is smaller than $\sigma_a(\boldsymbol{e})$ (3).

$$f_a(\boldsymbol{e}) = \{\boldsymbol{x} \mid d(\boldsymbol{x}, c(\boldsymbol{e})) \leq \sigma_a(\boldsymbol{e})\} \tag{3}$$

$$f_a(S) = \{\boldsymbol{x} \mid \exists \boldsymbol{e} \in S : d(\boldsymbol{x}, c(\boldsymbol{e})) \leq \sigma_a(\boldsymbol{e})\} \tag{4}$$

We extend the definition of f_a to range over sets S of σ-points (4). We observe:

- The projection function f_a maps any location to an n-dimensional region.
- A σ-point and the singleton set containing it are mapped to the same n-dimensional region in $\mathbb{R}^n$.
- A union of sets S_1, S_2 can map to a connected region $f_a(S_1 \cup S_2)$, without the set $S_1 \cup S_2$ being connected.

The third property is particularly relevant when we want to handle clusters, as clusters are collections of positions and/or regions. It reflects an often desirable property of clusters, to be able to represent space in terms of non-overlapping partitions (cf. [16]).

Taking the three properties together we conclude that positions, regions, and clusters can be interpreted as sets S of σ-points that are distinguished only by internal topological properties: positions are represented by singleton sets, regions by sets that fulfil additional topological criteria such as connectedness [13], and clusters by finite unions of sets.

4 Relations between Sets of $\boldsymbol{\sigma}$-Points

We can now specify the mereotopological and directional relations. For this discussion, we will take the perspective of a single application with a fixed accuracy a. Accordingly, we leave out the application accuracy index a in the following specifications, that is, write f and σ instead of f_a and σ_a and define o, p, EC, etc, instead of o_a, p_a, EC_a, etc.

4.1 Mereotopological Relations

We define functions o and p to express a degree of *overlap* o, *parthood* p, and *similarity* q between two sets of σ-points:

$$o(S_1, S_2) = \inf_{\boldsymbol{e_1} \in S_1, \boldsymbol{e_2} \in S_2} \sigma(\boldsymbol{e_1}) + \sigma(\boldsymbol{e_2}) - d(c(\boldsymbol{e_1}), c(\boldsymbol{e_2})) \tag{5}$$

$$p(S_1, S_2) = \sup_{\boldsymbol{e_1} \in S_1} \inf_{\boldsymbol{e_2} \in S_2} \sigma(\boldsymbol{e_2}) - \sigma(\boldsymbol{e_1}) - d(c(\boldsymbol{e_1}), c(\boldsymbol{e_2})) \tag{6}$$

$$q(S_1, S_2) = \max(p(S_1, S_2), p(S_2, S_1)) \tag{7}$$

$$\lim_{\epsilon \to 0} q(S_1, S_2) = d_H(f(S_1), f(S_2)) \tag{8}$$

For sets S_1, S_2 whose σ-points $\boldsymbol{e} \in S_1 \cup S_2$ are point-like, i.e. $\sigma(\boldsymbol{e}) < \epsilon$, the function q converges to the Hausdorff-distance d_H (8).

The RCC-relations [13] of *connection* C, *disconnection* DC, and *part-of* P between sets of σ-points S_1 and S_2 can be interpreted as relations between regions of $\mathbb{R}^n \times \mathbb{R}^+$:

$$\begin{aligned} C(S_1,S_2) &\text{ iff } o(S_1,S_2) \geq 0 & PO(S_1,S_2) &\text{ iff } o(S_1,S_2) > 0 \\ DC(S_1,S_2) &\text{ iff } o(S_1,S_2) < 0 & EC(S_1,S_2) &\text{ iff } o(S_1,S_2) = 0 \\ P(S_1,S_2) &\text{ iff } p(S_1,S_2) \geq 0 & EQ(S_1,S_2) &\text{ iff } q(S_1,S_2) = 0 \\ NTP(S_1,S_2) &\text{ iff } p(S_1,S_2) > 0 & TP(S_1,S_2) &\text{ iff } p(S_1,S_2) = 0 \end{aligned} \tag{9}$$

Further relations from [13] (PP, TPP, NTPP) can be defined from the above. Note that the σ-point relations based on p can diverge from corresponding relations over the regions projected by f for discontinuous sets S_1, S_2. As discussed above, clusters representing partitions must typically contain such discontinuities. If this is to be avoided in a given application, a scale-dependent hull operation such as the one proposed in [15] has to be employed.

4.2 Directional Relations

For modelling directional relations, we need not only σ-points but also higher-dimensional geometric entities, such as σ-lines and σ-planes. Like lines are defined by two different points, σ-lines can be defined by two σ-points, and σ-planes, by three σ-points. The σ-lines and σ-planes are crucial for interpreting directional relations. The concept of parallelism of directed axes and their quadrants in a plane is the core of reasoning with the directional calculus of Ligozat [10]. Any σ-line can be directed by two of its σ-points so as to give rise to two orderings ($>$ and $<$) between its σ-points and between σ-points on a parallel (10). Three σ-points, in turn, give rise to four different orderings ($<<, ><, >>, <>$) of a σ-plane and, with more than two dimensions, in parallel σ-planes (11):

$$\begin{aligned} \boldsymbol{e'} >_{\boldsymbol{e_0},\boldsymbol{e_1}} \boldsymbol{e} &\text{ iff } \exists \lambda > 0 : \boldsymbol{e'} = \boldsymbol{e} + \lambda(\boldsymbol{e_1} - \boldsymbol{e_0}) \\ \boldsymbol{e'} <_{\boldsymbol{e_0},\boldsymbol{e_1}} \boldsymbol{e} &\text{ iff } \exists \lambda < 0 : \boldsymbol{e'} = \boldsymbol{e} + \lambda(\boldsymbol{e_1} - \boldsymbol{e_0}) \end{aligned} \tag{10}$$

$$\begin{aligned} \boldsymbol{e'} >>_{\boldsymbol{e_0},\boldsymbol{e_1},\boldsymbol{e_2}} \boldsymbol{e} &\text{ iff } \exists \lambda_1 > 0, \lambda_2 > 0 : \boldsymbol{e'} = \boldsymbol{e} + \lambda_1(\boldsymbol{e_1} - \boldsymbol{e_0}) + \lambda_2(\boldsymbol{e_2} - \boldsymbol{e_0}) \\ \boldsymbol{e'} ><_{\boldsymbol{e_0},\boldsymbol{e_1},\boldsymbol{e_2}} \boldsymbol{e} &\text{ iff } \exists \lambda_1 > 0, \lambda_2 < 0 : \boldsymbol{e'} = \boldsymbol{e} + \lambda_1(\boldsymbol{e_1} - \boldsymbol{e_0}) + \lambda_2(\boldsymbol{e_2} - \boldsymbol{e_0}) \\ \boldsymbol{e'} <<_{\boldsymbol{e_0},\boldsymbol{e_1},\boldsymbol{e_2}} \boldsymbol{e} &\text{ iff } \exists \lambda_1 < 0, \lambda_2 < 0 : \boldsymbol{e'} = \boldsymbol{e} + \lambda_1(\boldsymbol{e_1} - \boldsymbol{e_0}) + \lambda_2(\boldsymbol{e_2} - \boldsymbol{e_0}) \\ \boldsymbol{e'} <>_{\boldsymbol{e_0},\boldsymbol{e_1},\boldsymbol{e_2}} \boldsymbol{e} &\text{ iff } \exists \lambda_1 < 0, \lambda_2 > 0 : \boldsymbol{e'} = \boldsymbol{e} + \lambda_1(\boldsymbol{e_1} - \boldsymbol{e_0}) + \lambda_2(\boldsymbol{e_2} - \boldsymbol{e_0}) \end{aligned} \tag{11}$$

For example, the directional relation system of cardinal directions north, north-east, east, etc. [10], can be described based on a reference system spanned by three σ-points $\boldsymbol{e_0}, \boldsymbol{e_N}, \boldsymbol{e_E}$ – where $\boldsymbol{e_N}$ is to the north of, and $\boldsymbol{e_E}$ to the east of, $\boldsymbol{e_0}$: the linear relations $>_{\boldsymbol{e_0},\boldsymbol{e_N}}$ (to the north of, or N), $<_{\boldsymbol{e_0},\boldsymbol{e_N}}$ (S), $>_{\boldsymbol{e_0},\boldsymbol{e_E}}$ (E), $<_{\boldsymbol{e_0},\boldsymbol{e_E}}$ (W) form the axes of the qualitative reference system, while the four planar relations $>>_{\boldsymbol{e_0},\boldsymbol{e_N},\boldsymbol{e_E}}$ (NE), $<>_{\boldsymbol{e_0},\boldsymbol{e_N},\boldsymbol{e_E}}$ (SE), $<<_{\boldsymbol{e_0},\boldsymbol{e_N},\boldsymbol{e_E}}$ (SW),

$><_{\boldsymbol{e_0},\boldsymbol{e_N},\boldsymbol{e_E}}$ (NW) form the quadrants. Note that the three σ-points $\boldsymbol{e_0}, \boldsymbol{e_N}, \boldsymbol{e_E}$ spanning the reference system need to have the same scale σ if we want to obtain a projection-based cardinal direction system, such as shown in Fig. 2.

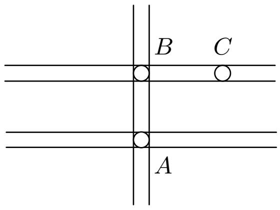

Fig. 2. Projection-based inference about three locations of the same size: C is to the East of B, and B is to the North of A.

The directional relations described above require positions $\boldsymbol{e}, \boldsymbol{e'}$. If we want to apply these relations to sets S, S', we need a transformation function that translates extended objects, i.e. regions and clusters, into positions. A transformation from arbitrary sets to positions can be achieved in a number of ways, e.g., by using a σ-point centred on the smallest enclosing sphere, where $e(\boldsymbol{x}, \sigma)$ is the function mapping a centre coordinate $\boldsymbol{x}$ given σ to coordinates $\boldsymbol{e}$ (12):

$$\mathrm{r2p}(S, \sigma) = e(c(\operatorname*{arg\,min}_{\boldsymbol{e} \in \mathbb{R}^n \times \mathbb{R}^+, P(S, \{\boldsymbol{e}\})} \sigma(\boldsymbol{e})), \sigma) \tag{12}$$

$$S' >^{\sigma}_{\boldsymbol{e_0},\boldsymbol{e_1}} S \text{ iff } \mathrm{r2p}(S', \sigma) >_{\boldsymbol{e_0},\boldsymbol{e_1}} \mathrm{r2p}(S, \sigma) \tag{13}$$

As *r2p* is a partial function, we conclude that directional relations are only applicable for scales σ on which *r2p* is defined. Definitions (10) and (11) can then be extended as exemplified in (13).

5 Summary and Conclusions

We presented an interpretation of mereotopological and directional relations for location models used in Ubiquitous Computing. Our approach respects a range of specific concerns in location modelling, such as uncertainty of location measurement, the mixture of qualitative and quantitative information and the specific types of representations employed in location models, namely, positions, regions, and clusters. We showed how all three representations can be interpreted in a unifying way using a scale space, and how applications can employ this representation to handle uncertainty from sensors and to ensure a required accuracy.

We gave a specification of mereotopological and directional relations in terms of this interpretation so as to allow integration of QSR-mechanisms into location models. Given this interpretation, relations can be computed from the quantitative parts of a location model and can be integrated with the qualitative parts so as to allow qualitative reasoning. The proposed specification is a first step towards an expressive, yet light-weight spatial reasoning mechanism for smart objects.

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